Hydrostatic Force on a Curved Surfaces

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1 Hydrostatic Force on a Curved Surface

On a curved surface the forces $p\delta A$ on individual elements differ in direction, so a simple summation of them may not be made. Instead, the resultant forces in certain directions may be determined, and these forces may then be combined vectorially. It is simplest to calculate horizontal and vertical components of the total force.

Horizontal component of hydrostatic force

Any curved surface may be projected on to a vertical plane. Take, for example, the curved surface illustrated in Fig. 1.

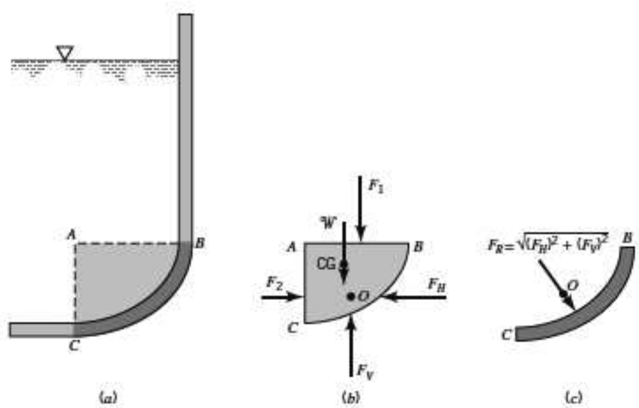

Figure 1: Hydrostatic force on a curved surface

Its projection on to the vertical plane shown is represented by the trace AC. Let F_x represent the component in this direction of the total force exerted by the fluid on the curved surface. F_x must act through the center of pressure of the vertical projection and is equal in magnitude to the force F on the fluid at the vertical plane.

In any given direction, therefore, the horizontal force on any surface equals the force on the projection of that surface on a vertical plane perpendicular to the given direction.

The line of action of the horizontal force on the curved surface is the same as that of the force on the vertical projection.

Vertical component of hydrostatic force

The vertical component of the force on a curved surface may be determined by considering the fluid enclosed by the curved surface and vertical projection lines extending to the free surface. Thus

$$F_H = F_2 = \rho g z_s \tag{1}$$
$$F_V = F_1 = \rho g V \tag{2}$$

where V is the volume of the liquid between the free surface liquid and solid curved surface. The magnitude of the resultant is obtained form the equation

$$F_R = \sqrt{F_H^2 + F_V^2} \tag{3}$$

Example 1 *A sector gate, of radius 4 m and length 5m, controls the flow of water in a horizontal channel. For the (equilibrium) conditions shown in Fig. 2 , determine the total thrust on the gate.*

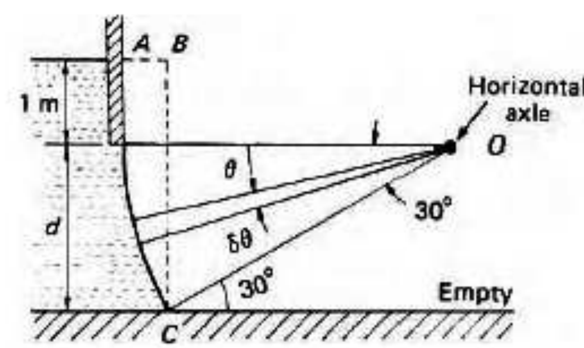

Figure 2: Hydrostatic force at a curved gate

Solution.

Since the curved surface of the gate is part of a cylinder, the water exerts no thrust along its length, so we consider the horizontal and vertical components in the plane of the diagram. The horizontal component is the thrust that would be exerted by the water on a vertical projection of the curved surface. The depth d of this projection is $4 \sin 30^o = 2$ m, ($R =$ 4 m) and its centroid is $1 + d/2 = 2$ m below the free surface. Therefore horizontal force F_H is equal

$$F_H = \rho g z_s A = 1000 \cdot 9.81 \cdot 2 \cdot (5 \cdot 2) = 1.962 \cdot 10^5 \ N$$

Its line of action passes through the center of pressure of the vertical projection, that is, at a distance I_x/Az_s below the free surface, given by:

$$\frac{I_x}{Az_s} = \frac{I_s + Az_s^2}{Az_s} = \frac{bd^3}{12(b\ d)z_s} + z_s = \frac{2}{12 \cdot 2} + 2 + 1.167 \ m$$

The vertical component of the total thrust is equal of weight of imaginary water ABC.

$$AB = (4 - 4\cos 30^o) = 0.536\ m$$

Vertical force is equal $F_V = \rho g V$. *Then*

$$F_V = 1000 \cdot 9.81 \cdot 5 \left[(0.536 \cdot 1) + \pi \cdot 4^2 \cdot \frac{30}{360} - \frac{1}{2} \cdot 2 \cdot 4 \cos 30^o \right] = 6.18 \cdot 10^4\ N$$

The horizontal and vertical components are co-planar and therefore combine to give a single resultant force of magnitude

$$|F| = \sqrt{F_H^2 + F_V^2} = (1.96210^5)^2 + (6.18104)^2 = 2.057 \cdot 10^5\ N$$

at angle equal to $arctg(\frac{6180}{196200}) \cong 17.5^o$ *to the horizontal axis.*

Problem 1.
The dam in Fig. 3 is a quarter circle 50 m wide into the paper. Determine the horizontal and vertical components of the hydrostatic force against the dam and the point CP where the resultant strikes the dam.

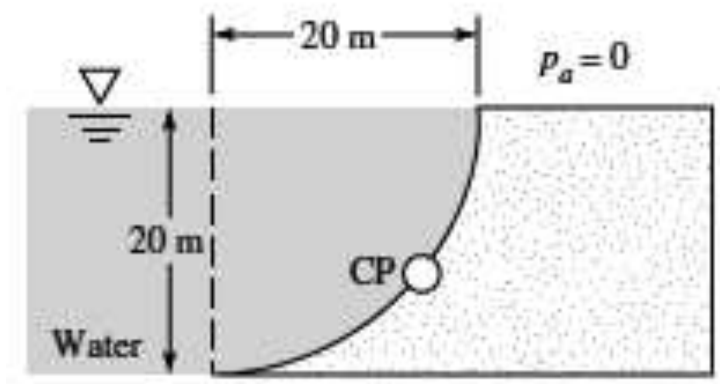

Figure 3: Hydrostatic force at a curved dam

(Ans. $F_H = 97.9\ MN, F_V = 153.8\ MN$)

Problem 2.
Compute the horizontal and vertical components of the hydrostatic force on the quarter-circle panel at the bottom of the water tank in Fig. 4

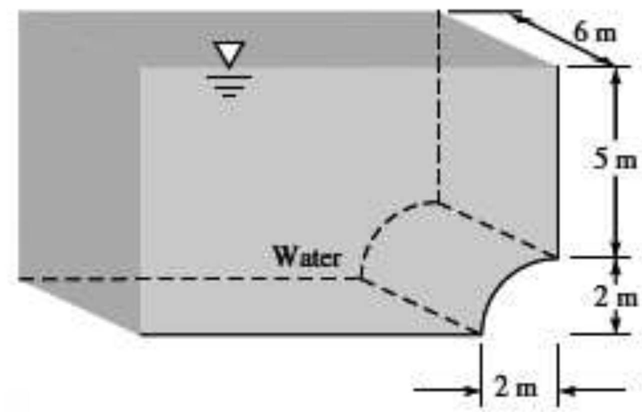

Figure 4: Hydrostatic force at the bottom of water tank

2 Buoyancy

Because the pressure in a fluid in equilibrium increases with depth, the fluid exerts a resultant upward force on any body wholly or partly immersed in it. This force is known as the **buoyancy**. This buoyancy force can be computed using the same principles used to compute hydrostatic forces on surfaces. The results are the two laws of buoyancy discovered by Archimedes in the third century B.C.:

1. a body immersed in a fluid experiences a vertical buoyant force equal to the weight of the fluid it displaces.

2. a floating body displaces its own weight in the fluid in which it floats.

So

$$\boxed{F_B = V_{body}\rho g} \tag{4}$$

In fig. 5 the vertical force exerted on an element of the body in the form of vertical prism of cross section δA is

$$\delta F = (p_2 - p_1)\delta A = \rho g h \delta A = \rho g \delta V$$

in which δV is the volume of the prism. Integrating over the complete body gives the formula (4).

Weighting an odd-shaped object suspended in two different fluids yields sufficient data to determine its weight W, volume V, specific weight$\gamma = \rho g$ (or density ρ) and specific gravity $S_G = \frac{\rho}{\rho_{water}}$. Figure 6 shows two free-body diagrams for the same object suspended and weighed in two fluids. F_1 and F_2 are the weight when submerged and $\gamma_1 = \rho_1 g$ and $\gamma_2 = \rho_2 g$ are the specific weights of the fluids. W and V, the weight and volume of the

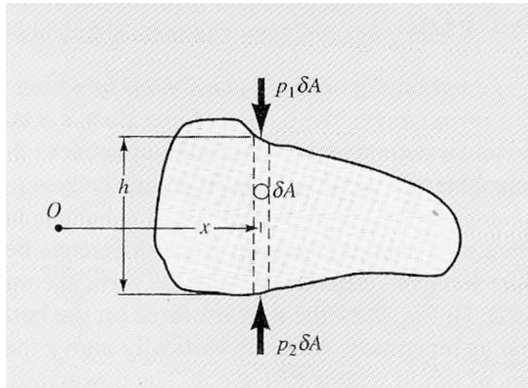

Figure 5: Vertical force componets on element of body

object, respectively, are to be found. The equations of equilibrium are written

$$F_1 + V\rho_1 g = W, \qquad F_2 + V\rho_2 g = W$$

and solved

$$V = \frac{F_1 - F_2}{g(\rho_1 - \rho_2)}, \qquad W = \frac{F_1\rho_1 - F_2\rho_2}{\rho_2 - \rho_1}$$

The hydrometr uses the principle of buoyant force to determine specific gravities of liquids.

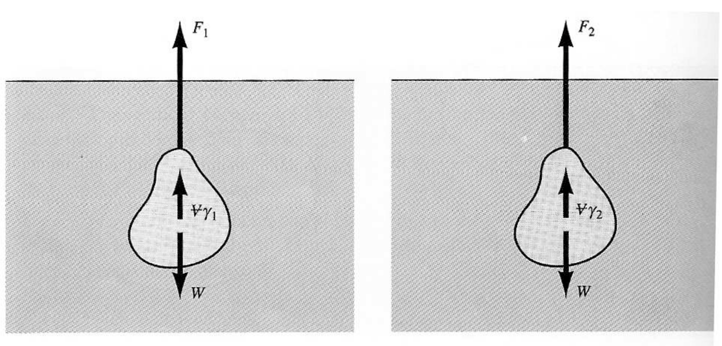

Figure 6: Free-body diagrams for body suspended in a fluid

Figure 7 shows a hydrometr in two liquids. It has a stem of prismatic cross section a. Considering the liquid on the left to be distilled water, $S_G = 1.00$, the hydrometr floats in equilibrium when

$$V_0 \rho g = W$$

in which V_0 is the volume submerged, $\rho g = \gamma$ is the specific weight of water, and W is the weight of the hydrometr. The position of the liquid surface is marked as 1.00 on the stem to indicate unit specific gravity S_G. When the hydrometr is floated in another liquid the equation of equilibrium becomes

$$(V_0 - \Delta V) S_G \rho g = W$$

in which $\Delta V = a\Delta h$. Solving for Δh using above equations gives

$$\Delta h = \frac{V_0}{a} \frac{S_G - 1}{S_G}$$

from which the stem can be marked off to read specific gravities or densities.

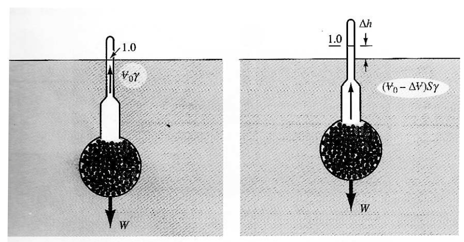

Figure 7: Hydrometr in water and in liquid of specific graviry S_G.

References

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