Fundamental Concepts Relating to Fluids

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1 Definition of a Fluids

A fluid is defined as a substance that deforms continuously whilst acted upon by any force tangential to the area on which it acts. Such a force is termed a ***shear force***, and the ratio of the shear force to the area on which it acts is known as the ***shear stress*** (see fig. 1). The characteristic that distinguishes a fluid from a solid is its inability to resist deformation under an applied shear stress (a tangential force per unit area). When a fluid is `at rest` neither shear forces nor shear stresses exist in it. A solid, on the other hand, can resist a shear force while at rest. In a solid, the shear force may cause some initial displacement of one layer over another, but the material does not continue to move indefinitely and a position of stable equilibrium is reached.

Definition 1 *Fluid is any substance that deforms continuously when subjected to a shear stress, no matter how small.*

Shear forces are possible only while relative movement between layers is taking place.

`Fluids` may be sub-divided into **liquids** and **gases**. A fixed amount of a liquid has a definite volume which varies only slightly with temperature and pressure. If the capacity of the containing vessel is greater than this definite volume, the liquid occupies only part of the container, and it forms an interface separating it from its own vapor, the atmosphere

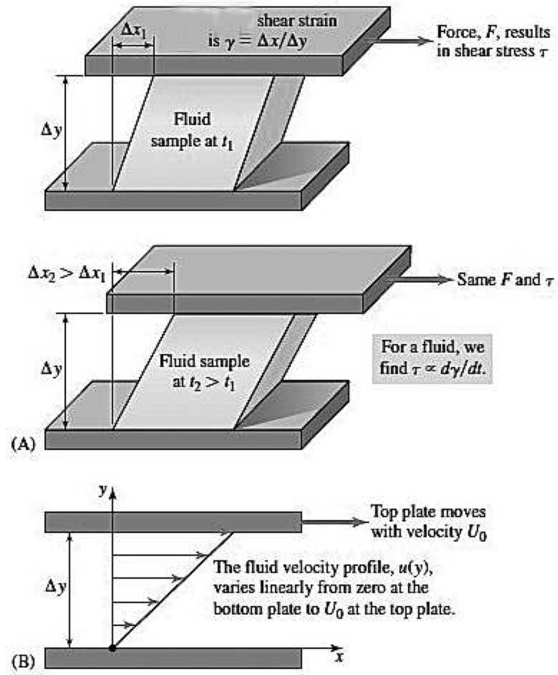

Figure 1: (A) The displacement and the corresponding shear strain increase linearly with time. For a fluid, the relationship between shear stress and shear strain is proportional. (B) The fluid velocity in the x direction, u, is a function of the y coordinate. The velocity $u(y)$ varies linearly from 0 at the bottom plate to U_0 at the top plate.

or any other gas present.

`Gas-` a fixed amount of a gas, by itself in a closed container, will always expand until its volume equals that of the container. Only then can it be in equilibrium.

In the analysis of the behavior of fluids an important difference between liquids and gases is that, whereas under ordinary conditions liquids are so difficult to compress that they may for most purposes be regarded as **incompressible**, gases may be compressed much more readily. Where conditions are such that an amount of gas undergoes a negligible change of volume, its behavior is similar to that of a liquid and it may then be regarded as incompressible. If, however, the change in volume is not negligible, the compressibility of the gas must be taken into account in examining its behavior.

Liquids have much greater densities than gases. As a consequence, when considering forces and pressures that occur in fluid mechanics, the weight of a liquid has an important role to play. Conversely, effects due to weight can usually be ignored when gases are considered.

The different characteristics of solids, liquids and gases result from differences in their

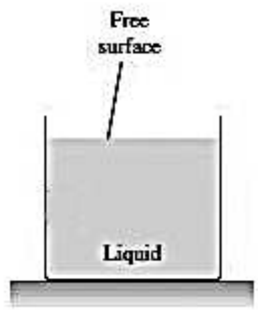

Figure 2: The liquid occupies only part of the container, and it forms an interface separating it from its own vapor.

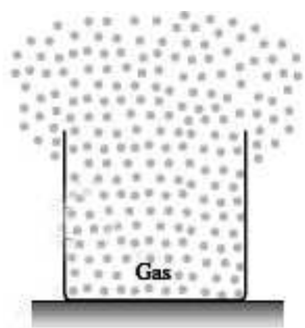

Figure 3: A fixed amount of a gas, by itself in a closed container, will always expand until its volume equals that of the container.

molecular structure. All substances consist of vast numbers of molecules separated by empty space. The molecules have an attraction for one another, but when the distance between them becomes very small (of the order of the diameter of a molecule) there is a force of repulsion between them which prevents them all gathering together as a solid lump.

1.1 Fluids as a continuum

The most fundamental idea we will need is the ***continuum hypothesis***. In simple terms this says that when dealing with fluids we can ignore the fact that they actually consist of billions of individual molecules (or atoms) in a rather small region, and instead treat the properties of that region as if it were a continuum. By appealing to this assumption we may treat any fluid property as varying continuously from one point to the next within the fluid; this clearly would not be possible without this hypothesis. To understand how physical quantities are defined in the continuum model we consider the following experiment to observe hob density of fluid is related to its molecular structure. At time t we consider a cube with the width α occupied by fluid centered at x_0.The average density of the fluid is $\rho_\alpha = M_\alpha/\alpha^3$, where M_α is the mass of the fluid inside of the cube. To define the density $\rho(x_0, t)$ it is examined what happens as α approaches zero. The graph fig. 4) shows the results. In region II, since there are many particles inside the cube, the average density ρ_α vary very little. If, however α were on the order of molecular distances, $\sim 10^{-9}$ meters, the may be only a few molecules in the cube and one will observe large fluctuations in ρ_α. Such rapid fluctuations are depicted in region I, It seems unreasonable therefore to define $\rho(x_0, t)$ as the limiting value of ρ_α as $\alpha \to 0$. Rather $\rho(x_0, t)$ should be defined as

$$\rho(x_0, t) = \lim_{\alpha \downarrow \alpha^*} \rho_{alpha}(x, t) \tag{1}$$

where α^* is the value of α where density begin to vary violently, here for example $\alpha^* = 10^{-9}$ meters. In a similar fashion other physical quantities can be considered as point functions in continuously distributed matter without regard to its molecular or atomic structure.

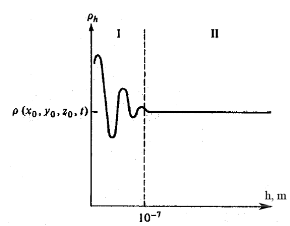

Figure 4: Graph of the average density ρ_α versus the with α.

2 Fluid properties

The properties of fluids permit us to distinguish one fluid from another, and they allow us to make estimates of physical behavior of any special fluid.

2.1 Density

Definition 2 *The density of a fluid (or any other form of matter) is the amount of mass per unit volume.*

$$\bar{\rho} = \frac{\Delta M}{\Delta V} \tag{2}$$

or the density at a point in fluid as

$$\rho = \lim_{\Delta \to 0} \frac{\Delta M}{\Delta V} \tag{3}$$

The unit of density is kg/m^3.

2.2 Pressure

A fluid always has pressure. As a result of innumerable molecular collisions, any part of the fluid must experience forces exerted on it by adjoining fluid or by adjoining solid boundaries. If, therefore, part of the fluid is arbitrarily divided from the rest by an imaginary plane, there will be forces that may be considered as acting at that plane. Pressure cannot be measured directly; all instruments said to measure it in fact indicate a difference of pressure. This difference is frequently that between the pressure of the fluid under consideration and the pressure of the surrounding atmosphere. The pressure of the atmosphere is therefore commonly used as the reference or datum pressure that is the starting point of the scale of measurement. The difference in pressure recorded by the measuring instrument is then termed the **gauge pressure**. The **absolute pressure**, that is the pressure considered relative to that of a perfect vacuum, is then given by

$$p_{abs} = p_{gauge} + p_{atm}.$$

The pressure is a scalar quantity (not vector!!. To say that pressure acts in any direction, or even in all directions, is meaningless. The SI unit of pressure is $N \cdot m^2$, now termed **pascal**, with the abbreviation Pa. Pressures of large magnitude are often expressed in atmospheres (abbreviated to atm). For precise definition, one atmosphere is taken as 1.01325105 Pa. A pressure of 10^5 Pa is called 1 bar. The thousandth part of this unit, called a millibar (abbreviated to mbar), is commonly used by meteorologists. It should be noted that, although they are widely used, neither the atmosphere nor the bar are accepted for use with SI units.

2.3 Viscosity

Viscous fluids tend to be gooey or sticky, indicating that fluid parcels do not slide past one another, or past solid surfaces, very readily. This can be an indication of some degree of internal molecular order, or possibly other effects on molecular scales; but in any case it implies a resistance to shear stresses. These observations lead us to the following definition.

Definition 3 *Viscosity is that fluid property by virtue of which a fluid offers resistance to shear stresses.*

At first glance this may seem to conflict with the earlier definition of a fluid (a substance that cannot resist deformation due to shear stresses), but resistance to shear stress, simply implies that the rate of deformation may be limited.
It is a matter of common experience that, under particular conditions, one fluid offers greater resistance to flow than another. Such liquids as tar, treacle,honey and glycerine cannot be rapidly poured or easily stirred, and are commonly spoken of as thick; on the other hand, so-called thin liquids such as water, petrol and paraffin flow much more readily. (Lubricating oils with small viscosity are sometimes referred to as light, and those with large viscosity as heavy; but viscosity is not related to density.) Gases as well as liquids have viscosity, although the viscosity of gases is less evident in everyday life.

Definition 4 ***Newton's Law of Viscosity - Newtonian fluids.*** *For a given rate of angular deformation of a fluid, shear stress is directly proportional to viscosity μ.*

$$\boxed{\tau = \mu \frac{dv}{dy}} \tag{4}$$

In fluid dynamics, many problems involving viscosity are concerned with the magnitude of the viscous forces compared with the magnitude of the inertia forces, that is, those forces causing acceleration of particles of the fluid. Since the viscous forces are proportional to the dynamic viscosity μ and the inertia forces are proportional to the density ρ, the ratio μ/ρ is frequently involved. The ratio of dynamic viscosity to density is known as the ***kinematic viscosity***

$$\boxed{\nu = \frac{\mu}{\rho}} \tag{5}$$

The SI unit for kinematic viscosity is $m^2 \cdot s^{-1}$.

2.4 Compressibility

All matter is to some extent compressible. That is to say, a change in the pressure applied to a certain amount of a substance always produces some change in its volume. Let us assume that to the volume $V(p_1, T)$, $T = const$, is a temperature) was applied the pressure $p_2 = p_1 + \Delta p$. Using the Taylor series we can write

$$V(p_1 + \Delta p) = V(p_1) + \frac{\partial V}{\partial p}\Delta p + 0(\Delta p^2) \tag{6}$$

The derivative $\frac{\partial V}{\partial p}$ devined by the intial volume $V(p_1$ is used as a measure of compressibility of liquids:

$$\boxed{\xi = -\frac{1}{V(p_1)}\frac{V(p_2) - V(p_1)}{p_2 - p_1}} \tag{7}$$

or in the differential form

$$\boxed{\xi = -\frac{1}{V(p_1)}\frac{\partial V}{\partial p}} \tag{8}$$

Since a rise in pressure always causes a decrease in volume, $\frac{\partial V)}{\partial p}$ is always negative, and the minus sign is included in the equation to give a positive value of ξ. Dimension of ξ is Pa^{-1}.
As the density ρ is given by mass/volume = m/V then

$$d\rho = d(\frac{m}{V}) = -\frac{m}{V^2}dV \tag{9}$$

so ξ may also be expressed as

$$\boxed{\xi = \frac{1}{\rho}\frac{\partial \rho}{\partial p}} \tag{10}$$

During this course I will be used the following books:

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