# Hydrodynamic lift force generated on the accelerated profile

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**Abstract.** In the paper we investigate the forces that act on the profile when profile starts suddenly to accelerate. It was proved that that sudden jump of acceleration cause sudden jump of the lift force. The peaks of force jumps do not depend on Reynolds number. To investigate that phenomena it was carry out the numerical simulation using the vortex particle method.

## 1. Introduction

In present work we examined the influence of the acceleration of the profile on the lift force generation in low Reynolds numbers flows, $Re < 10^4$. This problem applies to the study of non-stationary aerodynamic effects that are typical for flying birds and insects. Birds and insects generate the lift and the thrust force by the motion which is called the flapping motion. Due to fact that flapping motion undergoes changeable acceleration one can observed peaks in the lift force evolution [2, 3]. Similar effect is very clearly visible in sudden starts of the profile [8, 9]. In the initial time at $t = 0$ the flow over the profile is potential and the vorticity around the profile is equal zero. When the viscosity come to the action the large amount of the vorticity on the profile is generated. That is responsible for sudden increase of the lift force. Similar peaks of forces appear when the profile is suddenly accelerated. It is seems that effect is caused by not sufficient production of the vorticity at moment of acceleration. To study this phenomenon we perform the numerical calculations for the accelerating profile. We assume that the velocity of the profile start to growth linearly. The calculations were done also for the case of a sudden start of the profile. It had been investigated how the acceleration of the profile impact on the lift force distribution over the profile. For our study we choose the vortex particle method which allows for easily and direct analysis of vorticity fields evolution around the profile.

## 2. Vortex in Cell method

The Navier-Stokes equations in primitive variables (u,p) can be transformed to the Helmholtz equations for the evolution of the vorticity. The Helmholtz equations in the non-inertial reference frame have the form

$$\frac{\partial(\omega + 2\Omega_0)}{\partial t} + (\nabla\omega) \cdot \vec{u} = \nu\Delta\omega \tag{1}$$

$$\Delta\psi = -\omega \tag{2}$$

$$\vec{u} = \nabla \times (0, 0, \psi) \tag{3}$$

where $\Omega$ means the angular velocity of the profile. For our purposes, it was convenient to change the frame of reference in such a way, that profile would be at rest. That cause that boundary condition for the stream function far from the body with specified angular $\Omega$ and translational $U_0$ velocity is

$$\psi_\infty = U_0(y\cos(\alpha) - x\sin(\alpha)) - \frac{\Omega_0}{2}(x^2 + y^2). \tag{4}$$

A detailed description of solution of the Helmholtz equations in the moving reference frame can be found in [1]. In present paper the numerical calculation of the moving profile in fluid were carried out by the Vortex in Cell method (VIC). The velocity of the particles were calculated using the numerical grid. To better fit the numerical grid to the solid boundary we take advantage of the conformal transformation. By the conformal mapping the physical non-uniform flow region $(x, y)$ is replaced by the rectangular one $\xi, \eta$) where the calculations are carried out very efficiently. The following conformal transformation was applied (figure 1)

$$x + iy = \cosh(\xi + i\eta). \tag{5}$$

In new variables the equations of motion have the form

$$\frac{\partial \omega}{\partial t} + (\nabla \omega) \cdot \vec{u} = \frac{\nu}{J}\Delta\omega, \tag{6}$$

$$\Delta\Psi = -J\omega, \tag{7}$$

where $J$ denotes the Jacobian of the transformation. The velocity field is obtained according to the formula

$$u = \frac{1}{J}\frac{\partial \psi}{\partial \eta}, \quad v = -\frac{1}{J}\frac{\partial \psi}{\partial \xi}. \tag{8}$$

In the VIC method the continuous vorticity field is approximated with the discrete particles distribution. The flow region is covered with the numerical grid $h = \Delta\eta = \Delta\xi$. In every grid node, the particle with $\Gamma_j = \int_A \omega d\xi d\eta$ circulation are placed, where $A = h^2$, and

$$\omega(\xi, \eta) = \sum_p \Gamma_p \delta(\xi - \xi_p)\delta(\eta - \eta_p). \tag{9}$$

The equations of motion were solved using viscous splitting algorithm [5]. First, the inviscid fluid motion equation is solved

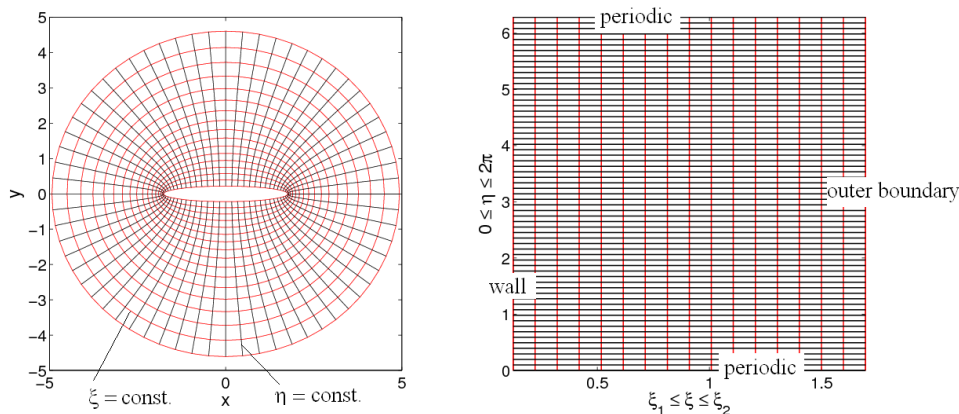$$\frac{\partial \omega}{\partial t} + (\nabla \omega) \cdot \vec{u} = 0. \tag{10}$$



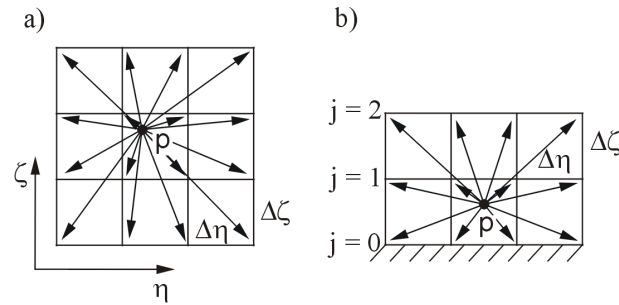**Figure 1.** Elliptical grid in the physical domain.

**Figure 2.** Redistribution of the particle masses onto the neighboring grid nodes, a) for particles laying inside of the computational domain (at least one cell from the wall), b) for the particles in the vicinity of the wall.

From (10) stems that vorticity is constant along the trajectory of the fluid particles. According to Helmholtz theorem [4, 5], vortex particles are moving like material fluid particles. The differential equation (10) is replaced by the infinite set of ordinary equations

$$\frac{d\xi}{dt} = u, \quad \frac{d\eta}{dt} = v, \quad \xi(0, \alpha_1) = \alpha_1, \quad \eta(0, \alpha_2) = \alpha_2. \tag{11}$$

where $\alpha = (\alpha_1, \alpha_2)$ means Lagrangian coordinate of fluid particles. The number of particles is equal to the number of grid nodes. The set of ordinary equations (11) were solved by the four order Runge-Kutta method. The velocity field was obtained by the solution of Poisson equation for the stream function and vorticity on the numerical grid (2). The velocities of the particles between the grid nodes were calculated by the two dimensional bilinear interpolation. In the second step the viscosity is taken into account by the solution of the diffusion equation

$$\frac{\partial \omega}{\partial t} = \frac{\nu}{J} \Delta \omega, \tag{12}$$

$$\omega(\xi, \eta, 0) = \omega_0, \quad \omega|_{wall} = \omega_s. \tag{13}$$

where $\omega_s$ denotes the vorticity at the solid boundary. The motion of the body in fluid and vorticity in the flow region generate non-zero tangent velocity component at the wall. The no slip condition is realized by adding the proper amount of vorticity at the wall. In present work we used the Briley formula of fourth order [6]. Nullifying of the normal velocity component at the wall was done by constant stream function at the wall $\psi = $ const. The diffusion equation was solved by alternating direction implicit method (ADI). In our calculations we use the numerical grid, therefore it's necessary to transfer the information from the vortex particles to the mesh nodes, figure 2. The redistribution of the vorticity from fluid particles to the mesh nodes was performed according to the formula

$$\omega_{ij} = \frac{1}{h^2} \sum_p \Gamma_p \varphi_h(\xi) \varphi_h(\eta). \tag{14}$$

where $\varphi_h(\cdot)$ denotes the interpolation kernel. The redistribution process plays a fundamental role for the accuracy of the vortex particle methods. In the current study we use the Z-splines interpolation functions [7] have a very good interpolation properties, especially when applied to vortex particle methods.
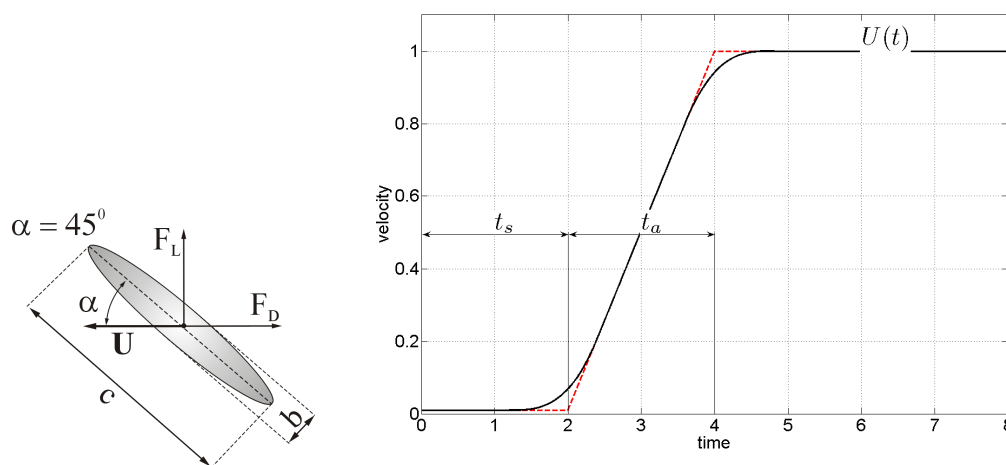
**Figure 3.** Computational parameters. Investigated profile on the left and change profile velocity.

## 3. Formulation of the problem and computation details

The numerical calculations were performed for a elliptic profile with relative thickness $e = b/c = 1/25$ and the angle of attack $\alpha = 45^0$. The flow configuration and parameters are shown on the left side of figure 3. The velocity of the profile $U$ is plotted on the right side of the figure. It is well known that for potential flow over the plate inclined to the direction of the incident flow a stagnation points appear on the upper and lower surface of the profile. Such a flow pattern cause the flow around the trailing edge from the lower to the upper surface. Theoretically an infinitely high velocity at the trailing edge cause an infinitely high negative pressure. By adding the circulation to the profile the rear stagnation point can be moved to the sharp trailing edge. In this way one can eliminate the flow around the trailing edge. In vortex particle method we begin the calculations from potential flow solution. In order to eliminate the flow over the trailing edge that cause also the generation of the large amount of the vorticity we decided to create the vorticity background around the profile. It was done by assuming at the beginning the small velocity around the profile $U = 0.01U_0$. The incremental velocity $U$ lasted a short interval time $t_s$. After the time $t_s$ the linearly growth of the velocity $U(t)$ to the nominal speed $U(t_a) = U_0$ occured. To obtain the smooth acceleration $dU/dt$, the curve $U(t)$ was smoothed in points ts and $t_s + t_a$ by polynomial of third order. The Reynolds number was defined based on the chord of the profile $c$: $Re = U_0c/\nu$ and the numerical calculations were performed in the range of Reynolds numbers 100 to 1000. The physical flow domain was transformed onto rectangular computational domain by the mapping (5), figure 1. The calculations were performed on a uniform grid of size: $512 \times 512$. The resolution of the numerical grid was chosen in such way that the further increasing of the mesh resolution does not affect the calculation results.

## 4. Computational results

The problem of moving profile in fluid belong to the category of external problems. In such a problems it is necessary to set the nominal velocity of the profile $U_0$. It is often done by the change of the profile velocity suddenly at the initial time: $U = 0$ for $t = 0$ and $U = U_0$ for $t > 0$. The sudden jump in the velocity of the profile from zero to the nominal velocity cause infinite increase of the lift force. In figure 4 it was shown the variation lift coefficient with time. Our numerical results was compared to the numerical results of [8]

The phenomenon of the high increase of the lift force at the initial time has been reported
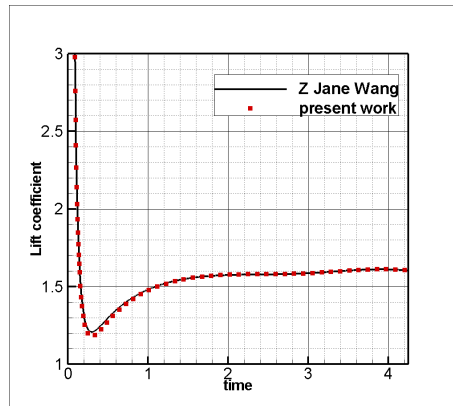
**Figure 4.** Lift coefficient versus time for sudden start of the profile, $Re = 1000$, angle of attack $\alpha = 45^0$.

repeatedly[8, 9]. In order to better investigate the this effect we performed a series of numerical tests where the profile was accelerated in different ways. To create the vorticity background around the profile during the time $t_s$ we assume small velocity of the profile: $U = 0.01U_0$. On figure 5 (on the left) the lift force for the three different start time $t_s$ is presented. The end of acceleration time $t_a$ was the same as well as the Reynolds number, $Re = 300$. We slightly smooth the acceleration curve around points $t_s$ and $t_a$. The presented test shows that the effect of jump in the lift force is not an numerical artifact and is connected with sudden acceleration of the profile. This effect is also independent of the length of the start time $t_s$ and of the existence of the background vorticity around the profile.

On the figure 6 the sequence of the stream function and vorticity field for the accelerated profile that correspond to the line $t_s = 4$ in figure 5 (line with number 3), is presented. The figure 6a correspond to the potential flow. It is visible the rear stagnation point on the upper surface of the ellipse. In the frame t = 3.7 (figure 6b) due to production of the background vorticity, one can see that the stagnation point was moved to the trailing edge. In the frame t = 3.8, after start of acceleration one can observe that stagnation point appears on the upper surface. Farther, this point was moved along the plate in upward direction. It correspond the sudden
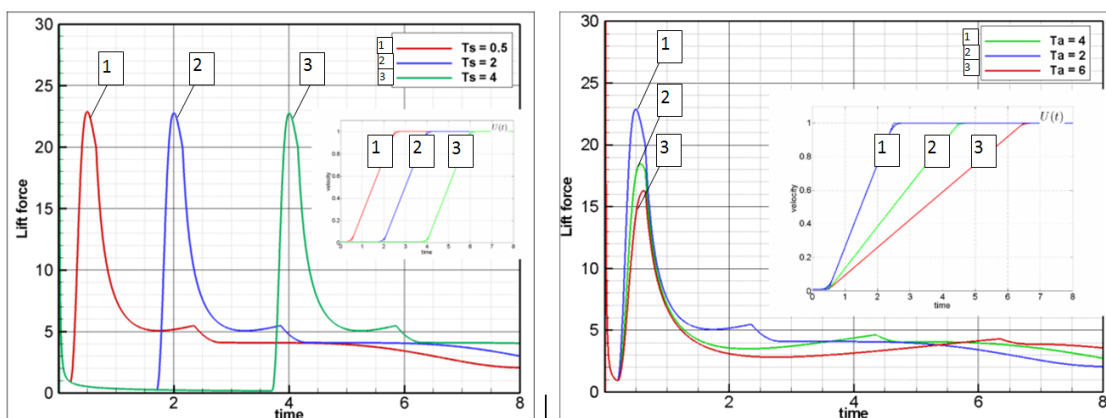


**Figure 5.** The effect of sudden start of the profile. On the figure the lift force for different start time $t_s$ on the left and with different acceleration time is presented.
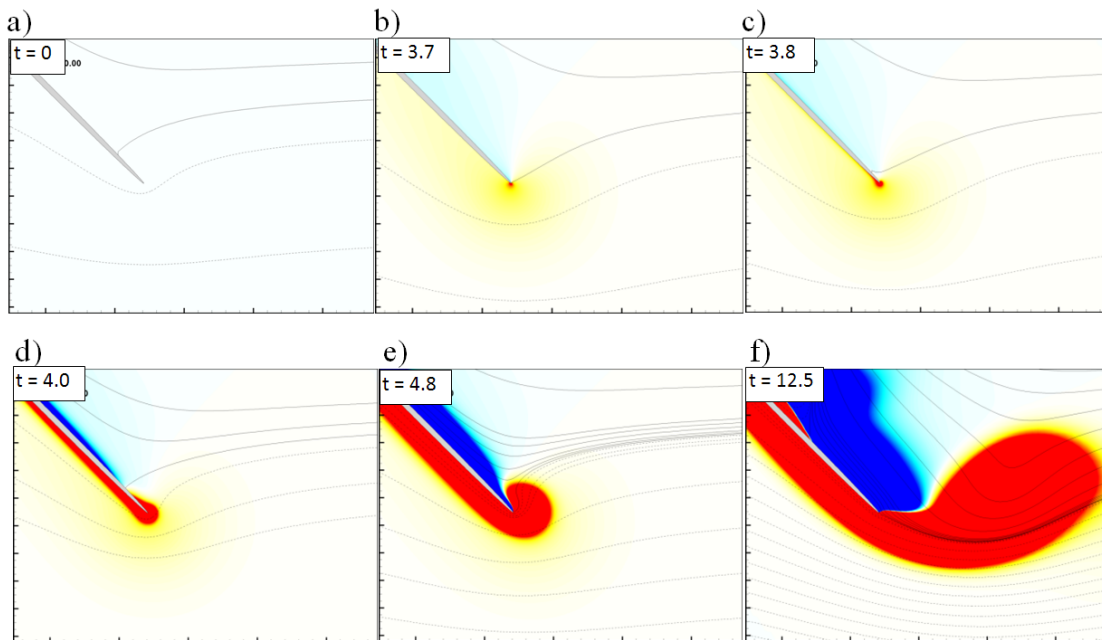
**Figure 6.** Vorticity contour and stream function over suddenly started profile.

increase of the lift force in figure 5. We believe that the effect of a sudden increase in lift force is related to the shift of the stagnation point on the upper edge of the profile.

We also check the influence the duration of acceleration time $t_a$ for the lift force. The calculations were done by selecting the $t_a = 2, 4$ and 6. figure 5 (on the right) shows plots of the calculated lift force. As we expected the greater velocity of acceleration causes the greater increase in the lift force. On the left of figure 7 the lift force for different way of smoothness of the $U(t)$ curve is presented. The curve with number 4 presents the acceleration of the profile without smoothing. One can see also that if the slope of the $U(t)$ is change more smoothly the increase with the lift force is reduced. On the right of figure 7 we presented the lift force of the accelerated profile for different Reynolds number. The peak in the lift force is the same for all examined Reynolds numbers. The increase in the lift force is the result of the inertia caused by the acceleration of
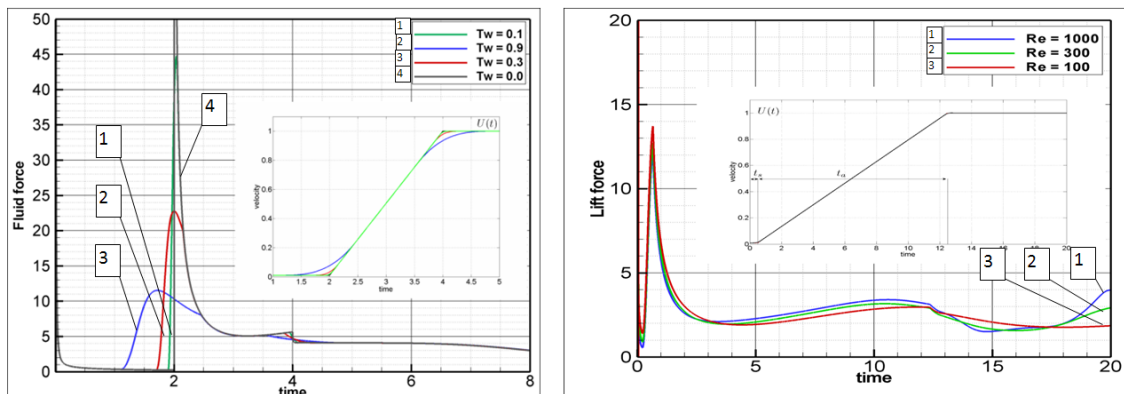


**Figure 7.** The lift force observed for sudden start of the profile. Influence of the Reynolds number on the left and influence of the Reynolds number on the right.
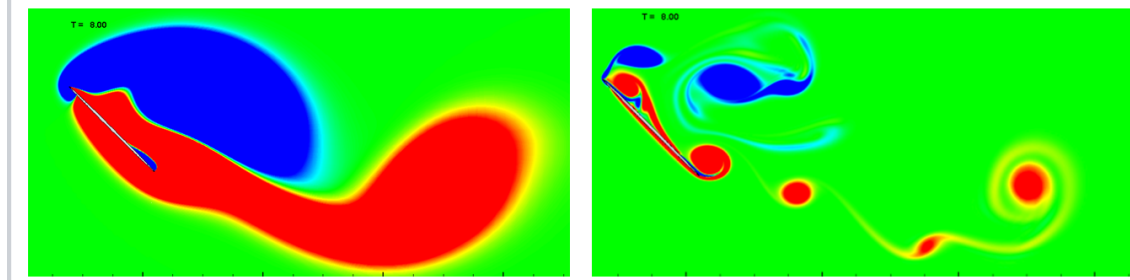
**Figure 8.** Vorticity contour around profile. On the left: $Re = 100$ and on the right: $Re = 1000$.

the profile. It worth notices that enlarge in the Reynolds number has effect on the reduction of the scale of the vortices around the body (see figure 8).

## 5. Conclusions

In the paper the effect of the acceleration of the profile was presented. It was found that acceleration caused sudden increase of the lift force. The effect of strong increase of the lift force is related with shift of the stagnation point from the trailing edge to the upper side of the profile. It has been shown that the peaks of the lift force do not depend on the Reynolds number. For the numerical simulation the Vortex in Cell method was used. The method is very well suited to study of the unsteady movement of body in the fluid, which is dominated by the vortex structures and it's interactions.

## 6. References

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