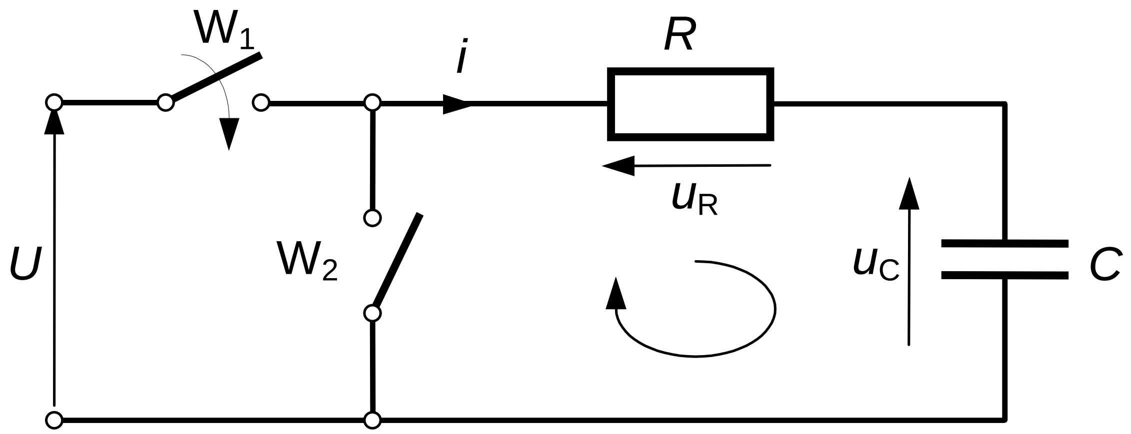
Stan przejściowy w gałęzi szeregowej *RC* przy wymuszeniu stałym

Ładowanie kondensatora



$$U = Ri + u_C \quad \Rightarrow \quad i = \frac{U - u_C}{R}$$

$$\mathrm{d}q = i\mathrm{d}t \wedge \mathrm{d}q = C\mathrm{d}u_C$$

$$\Downarrow$$

$$i = C\frac{\mathrm{d}u_C}{\mathrm{d}t}$$

$$RC\frac{\mathrm{d}u_C}{\mathrm{d}t} + u_C = U$$

Dla $U = 0$ — równanie różniczkowe liniowe jednorodne

$$T\frac{\mathrm{d}u_C}{\mathrm{d}t} + u_C = 0 \quad \Rightarrow \quad \frac{\mathrm{d}u_C}{u_C} = -\frac{\mathrm{d}t}{T}$$

gdzie $T = RC$ — stała czasowa obwodu

$$\ln u_C = -\frac{1}{T}\int_0^t \mathrm{d}t + A = -\frac{t}{T} + \ln A$$

$$u_C = Be^{-\frac{t}{T}}$$

Niech $B = B(t)$, zatem

$$\frac{\mathrm{d}u_C}{\mathrm{d}t} = \dot{B}e^{-\frac{t}{T}} - \frac{B}{T}e^{-\frac{t}{T}}$$

$$\dot{B}Te^{-\frac{t}{T}} = U$$

$$\Downarrow$$

$$B = U\int_0^t \frac{e^{t/T}}{T}\mathrm{d}t + D = Ue^{\frac{t}{T}} + D$$

$$u_C = \left(Ue^{\frac{t}{T}} + D\right)e^{-\frac{t}{T}} = U + De^{-\frac{t}{T}}$$

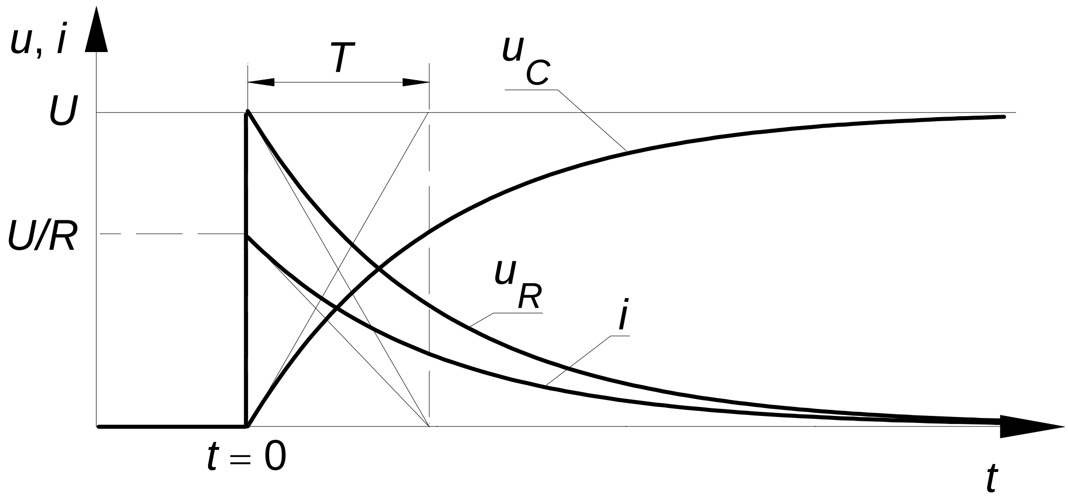
Warunek początkowy: $t = 0 \;\Rightarrow\; u_C(0) = 0$

$$0 = U + D \quad\Leftrightarrow\quad D = -U$$

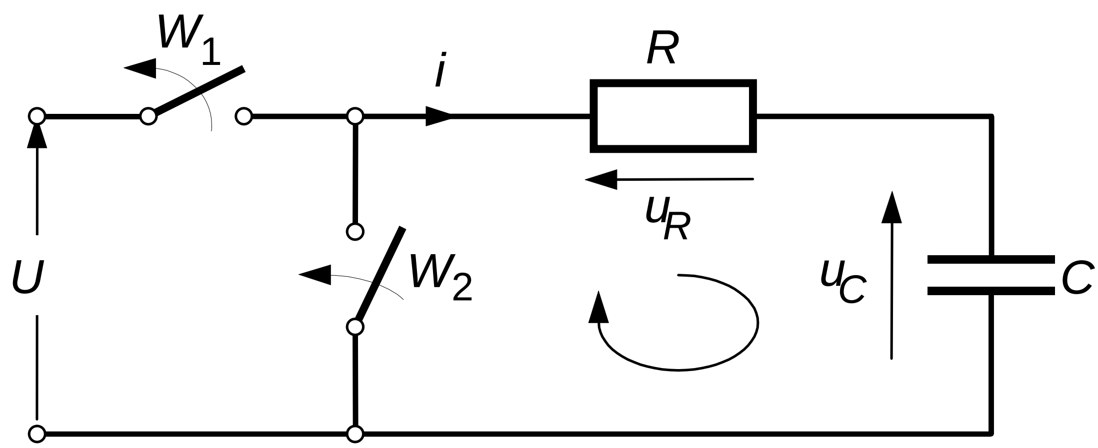
$$u_C = U - Ue^{-\frac{t}{T}} = U\left(1 - e^{-\frac{t}{T}}\right)$$

$$t \to \infty \quad\Rightarrow\quad u_C \to U$$

$$i = C\frac{\mathrm{d}u_C}{\mathrm{d}t} = CU\frac{\mathrm{d}}{\mathrm{d}t}\left(1 - e^{-\frac{t}{T}}\right) = \frac{C}{T}Ue^{-\frac{t}{T}} = \frac{U}{R}e^{-\frac{t}{T}}$$



Rozładowanie kondensatora



$$u_C + Ri = 0 \quad \Rightarrow \quad u_C = -Ri = -RC\frac{\mathrm{d}u_C}{\mathrm{d}t}$$

$$\frac{\mathrm{d}u_C}{u_C} = -\frac{\mathrm{d}t}{T} \quad \Rightarrow \quad \ln u_C = -\frac{t}{T} + A$$

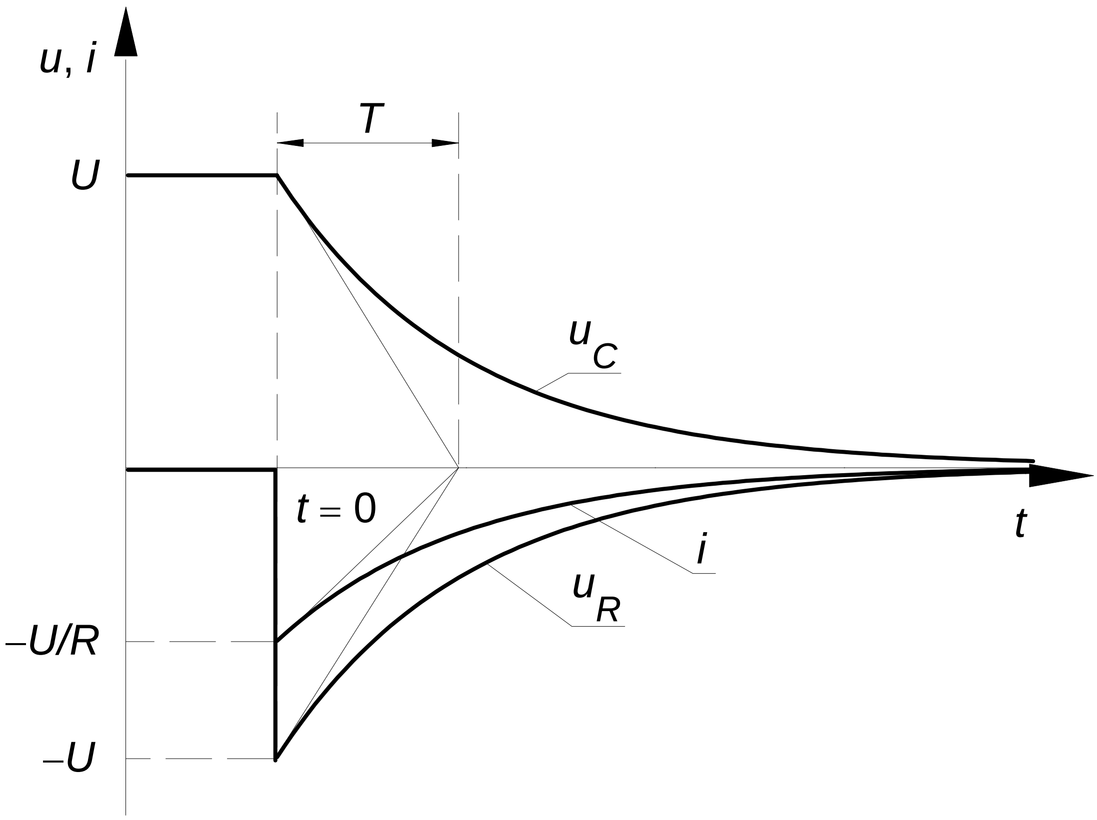
$$u_C = Be^{-\frac{t}{T}}$$

Warunek początkowy: $t = 0 \;\Rightarrow\; u_C(0) = U$

$$B = U$$

$$u_C = Ue^{-\frac{t}{T}}$$

$$i = -\frac{u_C}{R} = -\frac{U}{R}e^{-\frac{t}{T}}$$



Energia wydzielona na rezystorze

$$W = \int_0^\infty Ri^2 \mathrm{d}t = \int_0^\infty R\left(-\frac{U}{R}e^{-\frac{t}{T}}\right)^2 \mathrm{d}t = \frac{1}{2}CU^2 = \frac{1}{2}Cu_C(0)^2$$